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Redefining the Kilogram: How and Why?

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Abstract

A redefinition of most of the base units of the International System (SI) has been proposed and may be approved as early as autumn, 2011. In particular, it is proposed that the kilogram, which currently is defined by the mass of an artifact maintained at the BIPM, should be redefined in terms of a fundamental constant of physics. The paper reviews the history of the present artifact definition, the evidence that a redefinition is desirable and the reason that electrical metrologists wish to redefine the kilogram in terms of the Planck constant. The fact that electrical metrology will greatly influence the choice of a definition of the mass unit is interesting and will be explained. Also explained is why, from the point of view of mass metrology, the choice of a particular defining constant over the other candidates has little relevance.

1. Introduction

Mass metrologists are generally aware that moves are afoot to redefine the kilogram and other base units of the International System (SI) [1] in terms of a set of physical constants with fixed values. The proposed new system is sometimes loosely referred to as the "quantum SI" because the preferred constants are important in the theories of quantum mechanics. The following paper focuses on how such changes will affect mass metrology and why, since about 1990, the kilogram definition has increasingly complicated other areas of science and metrology.

We begin with a brief history in the SI kilogram from its origin to about 1990 and then proceed to discuss the present situation, which largely developed after 1990. We conclude with the prospects for redefining the kilogram in 2011 and the steps that mass metrologists are taking to prepare for this eventuality.

2. The Definition of the Kilogram and its History until 1990

2.1 *Early History*

The history of the present definition of the kilogram is well known and documented [2]. Nevertheless, several important features of the definition are important to the following discussion and will, therefore, be restated.

First and most important, the definition of the kilogram is very simply stated:

"The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram" [1].

This definition dates from 1901 but merely refines the first definition of 1889. As is well known, the international prototype of the kilogram (IPK) is made of an alloy of 90% platinum and 10% iridium in the form of a cylinder with height (39 mm) approximately equal to diameter.

In the 21st century, the key feature of the kilogram definition is that it is based on the mass of a single artifact stored at the International Bureau of Weights and Measures (BIPM), and it is the only base unit still defined by an artifact. Almost 100 copies of the IPK have been manufactured. Six of these, the official copies, are stored with the IPK in a vault at the BIPM[1]. Eight are used by the BIPM for routine calibration work and for research. The remaining artifacts have been distributed as requested to Member States of the BIPM to serve as national prototypes. In essence, every mass measurement whose result is expressed in kilograms must be traceable to the IPK.

Since it was placed in service in 1889, the IPK has only been used during two measurement campaigns, known as the second and third periodic verifications of national prototypes of the kilogram. The third verification was carried out in 1989-1992 and the results tell us most of what we know about the IPK and its copies [3].

The comportment of the official copies with respect to the IPK is shown in a well-known graph (Fig. 1). This graph shows the *change* in calibrated mass with respect to time of each copy compared with its initial calibration. Thus the horizontal axis represents the mass of the IPK and each of the curves starts on the horizontal axis at the date of its first calibration traceable to the IPK. Similar graphs would be obtained for most of the other copies. Over a period of about 100 years, it is seen that the masses of the official copies tend to be increasing (with respect to the mass of the IPK) at a rate of approximately 0.5 µg/year. Note that the trend was already evident after the second verification, which ended in about 1950.

This behaviour, difficult to study but certainly not due to any unknown physical process, is a defect of artifacts. Indeed, perhaps the masses of all Pt-Ir 1 kg artifacts are drifting with respect to the fundamental constants. (An effect that is common to each member of a group of standards can never be detected by measuring differences between group members.) It has thus been obvious since about 1950 that mass metrology would benefit from a standard linked to a constant of nature. The mass of an atom of carbon-12, $m(^{12}\mathrm{C})$ was an obvious candidate because ^{12}C forms the basis of the atomic mass scale, where the atomic mass unit, or dalton, is defined as $\mathrm{u} = m(^{12}\mathrm{C})/12$. The relative atomic mass $A_r(X)$ of an atom X is defined as $m(X)/\mathrm{u}$. Note that, $A_r(^{12}\mathrm{C}) = 12$ (exactly). Relative atomic masses are useful because the uncertainty with which they can be measured is much smaller that the uncertainty of u itself, which depends on a link to the IPK. The large uncertainty in u reflects the difficulty in measuring $\mathrm{m(IPK)/m(^{12}C)} \sim 10^{26}$ to high accuracy. The fact that the IPK is an artifact and might not be stable with respect to the fundamental constants to better than, say, 5 µg/year is an additional, though until very recently, a secondary problem.

2.2 *Consequences of the third Verification of National Prototypes of the Kilogram*

Even before the final report of the third verification confirmed the trends shown in Fig. 1, initial results were already known to the international community of mass metrologists. As a consequence, Davis estimated the stability of the mass of the IPK with respect to physical constants but the uncertainty of the available data was insufficient to draw any useful conclusions [4]. Quinn suggested that national metrology institutes (NMIs) seek to improve the uncertainty of experiments designed to link the IPK to a fundamental constant [5]. A target uncertainty[2] of 20 µg was suggested because that uncertainty would permit useful monitoring of the stability of the artifact kilogram. For instance, over a period of ten to twenty years it would be possible to determine the stability of the IPK with respect to a constant of nature and, in due course, to redefine the SI unit of mass. Similar sentiments were expressed by the Consultative Committee for Mass and Related Quantities (CCM), eventually resulting in Resolution 5 of the 20th General Conference of Weights and Measures (CGPM), 1995.

Even as this strategy was being formulated, other communities of metrologists became impatient to redefine the kilogram. Their increasingly strong objections to the comparatively relaxed approach of Resolution 5 are not always appreciated by the mass community.

3. What has Changed Since 1990

3.1 *Quantum Electrical Standards*

During the last 20 years, the picture has changed, and this is due to the discovery, understanding and perfection of quantum electrical standards. Recall that the base unit of electricity, the ampere, is defined in

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[1] A photograph of the international prototype and its six official copies can be found at http://www.bipm.org

[2] Throughout this paper we use standard uncertainties (coverage factor k=1).

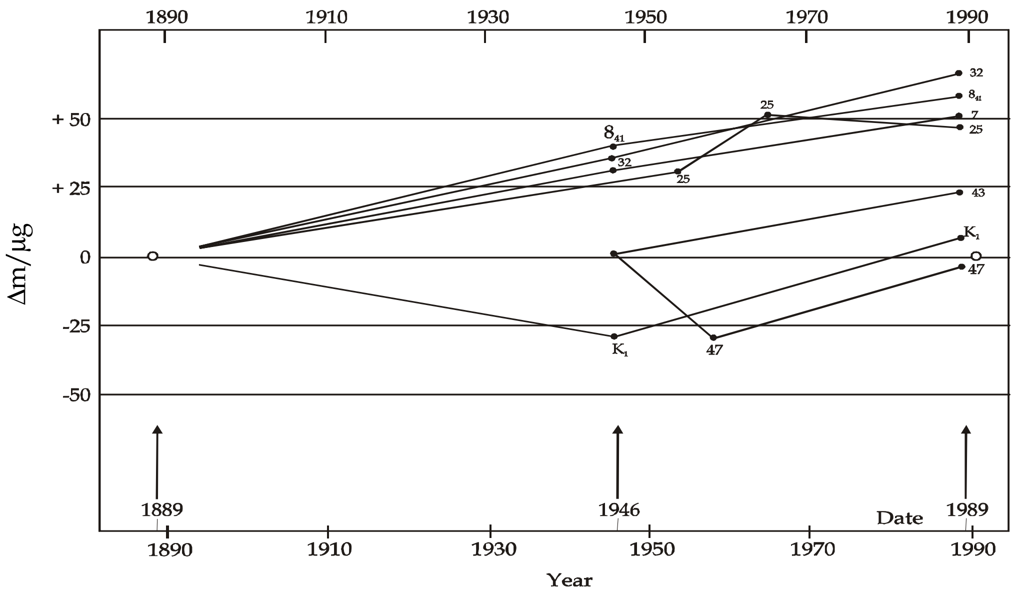


Fig. 1. Behaviour of the six official copies and prototype No. 25 with respect to the international prototype. Copies No. 43 and 47 received their first calibrations in 1946 at the start of the second verification

terms of the force per unit length of two idealized, parallel wires carrying identical current [1]. The details of this definition fix the value of μ_0, the magnetic constant, to be $4\pi \times 10^{-7}$ N/A^2 exactly. From basic physics, we know that $c_0^2\mu_0\varepsilon_0 = 1$, where c_0 is the speed of light in vacuum, now a fixed number, and thus ε_0, the electric constant, is also fixed. These constants have been used to link practical electrical standards to the SI by means of devices such as the ampere balance and the calculable capacitor.

With the adoption of quantum electrical standards [6], it became possible to measure a d.c. voltage U in terms of an accurately known microwave frequency f through the relation

$$U = nK_J^{-1} f \qquad (1)$$

where n is an integer selected by the metrologist and K_J is the Josephson constant. The latter is now generally accepted to be exactly 2e/h, where e is the elementary charge and h is the Planck constant.

A second quantum standard is now used to produce a known resistance R_H by means of the quantum Hall effect. In this case,

$$R_H = R_K/i \qquad (2)$$

where i is a small integer selected by the metrologist and R_K is the von Klitzing constant, generally accepted to be exactly h/e^2.

Measurement results obtained from a variety of Josephson and quantum Hall devices have been compared to very high precision and appear to be identical [6]. Consequently, electrical metrologists routinely carry out measurements to a precision far greater than accuracy with which either 2e/h or h/e^2 is known in SI units. The problem is that in the SI, experimental determinations of h and e are made relative to a macroscopic artifact, the IPK. We will discuss the measurement of h below. At this point we note that the units of h, joule-seconds, clearly involve the kilogram. The elementary charge e is derived from the relation

$$\alpha = \frac{\mu_0 c_0 e^2}{2h} \qquad (3)$$

Recall that c_0 and μ_0 have defined SI values, and therefore no uncertainty. The dimensionless constant, α, known as the fine-structure constant, has recently been measured with relative uncertainty less than 1×10^{-9}. (The value of the fine-structure constant is independent of unit systems and may not be fixed arbitrarily [7].) Thus the measure of e is correlated with that of h. Here and in the sequel, we rely heavily on the 2006 CODATA compilation [8] as the authoritive reference on fundamental constants expressed in the SI.

It is difficult to measure h in terms of the IPK and thus the lowest uncertainty that has been attained, 50×10^{-9} according to [8], is still greater than the precision with which electrical metrologists can measure voltage and resistance using quantum electrical devices. In addition, it is inconvenient for metrologists to re-evaluate their results every time a new value of h is recommended by CODATA. This situation led to the adoption in 1990 of conventional values for the Josephson and von Klitzing constants. Of course the choice of conventional values was guided by the best available SI values at that time. However, conventional values have no uncertainty; the voltage and resistance calibrations derived from these values are merely close approximations to SI values. An obvious solution, pleasing to electrical metrologists, would be to eliminate the use of conventional values for the Josephson and von Klitzing constants. The preferred approach [9] is a) to fix the SI value of the Planck constant so that it is no longer measured in terms of the IPK; b) to fix the SI value of the elementary charge so that the values of the Josephson and von Klitzing constants expressed in SI have no uncertainty; c) to change μ_0 to a derived constant. The last step is required so that the fine-structure constant (Eq. 3) remains an experimentally-determined quantity, as it must. Of course the effects of a) and b) are to redefine the kilogram in terms of the Planck constant, the metre and the second (exact wording to be determined) and to redefine the ampere in terms of the elementary charge and the second.

3.2 Fundamental Constants

Two classes of experiments provide independent methods to link the artifact kilogram to various constants of physics, including the Planck constant h, the Avogadro constant N_A, and the atomic and subatomic masses. At present, these experiments provide the SI values of the constants in question. This means that many fundamental constants of physics are being measured in terms of an artifact manufactured in the late 19th century. Since the artifact kilogram can have no bearing on basic physics, correlations due to traceability to the IPK are an annoyance to many physicists. We have already seen that removal of the correlation between atomic masses and the IPK was accomplished *ad hoc* by the introduction of the (non SI) atomic mass unit, u. The current situation can be illustrated by looking at the variance-covariance matrix of the relative uncertainties of four selected constants whose measured values require traceability to the IPK (Table 1). Note that the absolute correlation coefficients

Table 1

Variance/Covariance/Correlation matrix of four fundamental constants of physics whose SI values involve traceability to the IPK. Relative variances and covariances are shown in bold, where each has been multiplied by 10^{16}. Correlation coefficients are shown in italics. This is taken from Table L of [8]

	h	**e**	**m_e**	**N_A**
h	**24.8614**	**12.4308**	**24.8611**	**-24.8610**
e	0.9999	**6.2166**	**12.4259**	**-12.4259**
m_e	0.9996	0.9992	**24.8795**	**-24.8794**
N_A	-0.9996	-0.9991	-1.0000	**24.8811**

Table 2

Same as Table 1 except that h and e have become fixed constants with no uncertainty. Fixing h implies redefining the kilogram and fixing e implies redefining the ampere

	h	**e**	**m_e**	**N_A**
h	**0**			
e		**0**		
m_e			**0.0185**	**-0.0184**
N_A			-0.956	0.0202

are almost unity in all cases (the coefficient given as -1.0000 is not exactly -1). In Table 2, we show the effect of defining fixed SI values of h and e, in similarity to the 1983 redefinition of the metre, which fixed the value of c_0, the speed of light in vacuum. The authors of [8] refer to this procedure of defining the values of a judiciously chosen set of physical constants as "a significant advance in our knowledge of the constants".

3.3 Links to the IPK

Without improved measurements, repartition of uncertainties as decided by committees is a zero sum game. If one wishes to fix the value of h, its present uncertainty would then be assigned to the IPK. Let us now examine the strongest links between the IPK and the fundamental constants in more detail.

3.3.1 The watt balance

The watt balance provides an experimental method to link a macroscopic mass standard to a constant of nature. There are many review articles that provide details of the design and operation of this type of device [10]. At the moment there are two operational watt balances that have produced complete results, published in peer-reviewed journals. These are located at the National Physical Laboratory (NPL, UK) and the National Institute of Standards and Technology (NIST, USA). However, other watt balances are being constructed at the national metrology institutes of Switzerland, and France, as well as at the BIPM. In addition, incremental improvements are being made to the NIST and NPL devices, although the latter is scheduled to cease operation at the end of 2008. Although these are complicated devices, their essence is to provide the experimental equivalence between mechanical power, measured in terms of the SI units kg, m and s, with electrical power measured in terms of non-SI conventional values of current and voltage (hence the name "watt" balance).

The final equation describing the watt balance is:

$$\mathrm{Mgv} = \mathrm{I}_{90}\mathrm{U}_{90} = \frac{\mathrm{U}'_{90}}{\mathrm{R}_{90}}\mathrm{U}_{90} \tag{4}$$

On the left-hand side of Eq. 4, M is the mass of a macroscopic object (traceable to the mass of the IPK), g is the local acceleration of gravity and v is a velocity (both g and v are traceable to the SI definitions of length and time). On the right-hand side, the current I_{90} and the voltage U_{90} are measured in terms of quantum electrical standards, where one has adopted the 1990 conventional values for the Josephson and von Klitzing constants, K_{J-90} and R_{K-90}. However, we can easily rewrite Eq. 4 so that the electrical power shown on the right-hand side is the SI value. The conventional "unit" of voltage is related to the SI unit by the ratio K_{J-90}/K_J; K_{J-90} is simply a conventional value with no uncertainty, but $K_J = 2e/h$. Similarly, the conventional "unit" of resistance is related to the SI unit by the ratio R_K/R_{K-90}; R_{K-90} is a conventional value with no uncertainty, but $R_K = h/e^2$. Thus Eq. 4 becomes:

$$\mathrm{Mgv} = \frac{\mathrm{h}}{4}\left(\frac{\mathrm{U}'_{90}}{\mathrm{R}_{90}}\mathrm{U}_{90}\right)\mathrm{K}^2_{\mathrm{J\text{-}90}}\mathrm{R}_{\mathrm{K\text{-}90}} \tag{5}$$

and we see that Eq. 5 gives us the possibility of determining the Planck constant, h, in SI units. We also see that in the present SI, this important constant- the fundamental constant of quantum mechanics-is measured in terms of the IPK.

3.3.2 X-ray crystal density method

A different and independent method can also be used to link a macroscopic mass standard (with mass traceable to the IPK) to a fundamental constant. First we show conceptually how a macroscopic mass standard can be linked to a particular atomic mass. If we have a macroscopic quantity, say 1 kg, of the isotope ^{28}Si in the form of a perfect single crystal, then its density, ρ (^{28}Si), will be homogeneous. Following [8], the mass *m*(^{28}Si) of a single silicon atom within the crystal is simply:

$$\mathrm{m}\left({}^{28}\mathrm{Si}\right) = \frac{\mathrm{a}^3}{\mathrm{n}}\rho\left({}^{28}\mathrm{Si}\right) \tag{6}$$

where a is the length of a unit cell of the crystal and n is the number of silicon atoms in this cell (in fact, n = 8). The length, a, is measured by means of an x-ray interferometer whose output is directly traceable to the metre. The density ρ (^{28}Si) is determined for the macroscopic sample by measuring its mass, M (traceable to the IPK) and its volume, V, traceable to the metre. Rearranging terms gives us:

$$M = m\left({}^{28}\mathrm{Si}\right)\frac{V}{a^3/n} \tag{7}$$

Because the masses of many stable isotopes, such as ^{28}Si, and subatomic particles, such as the electron, are known to very high accuracy relative to the mass of ^{12}C, we may think of this experiment as linking the kilogram to the mass of ^{12}C or to the mass of a single electron. The importance of m(^{12}C) has already been discussed in Section 2.1. For example, the relative atomic mass of ^{28}Si, defined by Eq. 8.

$$A_r\left({}^{28}\mathrm{Si}\right) = 12\frac{m\left({}^{28}\mathrm{Si}\right)}{m\left({}^{12}\mathrm{C}\right)} \tag{8}$$

is already well known and we may therefore rewrite Eq. 7 by substituting m(^{12}C)A_r(^{28}Si)/12 for m(^{28}Si) without any significant increase in experimental uncertainty.

The SI base unit "mole" is defined as "the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of ^{12}C" [1], and the Avogadro constant, N_A, is defined as the number of entities per mole of a specified substance. For the substance ^{12}C, it is therefore evident that N_A = (0.012)/m (^{12}C). Thus, after writing Eq. 7 in terms of the mass of ^{12}C, we may rewrite it again in terms of the Avogadro constant, with no increase in uncertainty. This approach to determine N_A is called the x-ray crystal density method, or XRCD method. The work is being carried out by a world-wide collaboration of partners, known as the International Avogadro Coordination. As with the watt balance, the experiment is very difficult to perform to high accuracy and involves many controls and corrections. Excellent review papers of this work are available [11].

We thus see that Eq. 7 can be used to link the kilogram to virtually any atomic mass or to the Avogadro constant. It is, perhaps, surprising that a useful relation exists to link the Planck constant to atomic masses, as we now discuss.

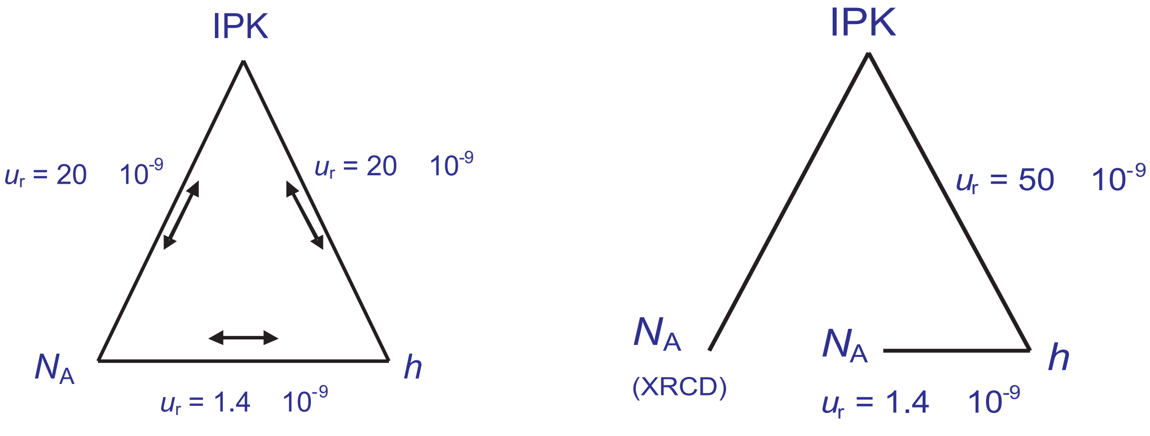
4. Consistency between Methods

In the CODATA report [8], it is shown that the product $N_A h$ and hence the quantity h/m(X), where X is a specified atomic isotope, are related in a simple way to a collection of fundamental constants having very small experimental uncertainties. The present relative uncertainty for $N_A h$ has been evaluated by CODATA to be less than 2×10^{-9} and this result is completely independent of watt balance or XRCD measurements.

There are two important consequences to having an uncertainty that is an order of magnitude below the present target uncertainties for watt balances and the Avogadro project. First, we may just as well think of a watt balance as measuring the Avogadro constant or the mass of a single electron in terms of the IPK; equivalently, we may think of the Avogadro experiment as determining the Planck constant in terms of the IPK. The added uncertainty is negligible in the present SI.

Second, we can use the fact that we know the value of $N_A h$ to high accuracy as a means to check the consistency of experimental results from watt balances (usually reported as values of h) and results obtained by the Avogadro project (usually reported as values of N_A). For instance, this is how the authors of the CODATA report [8] are able to show a graph of various determinations of h which nevertheless includes results from the Avogadro project and a measurement of the Faraday constant (eN_A).

The consistency check may be visualized as a triangle whose vertices are the IPK, the Planck constant determined by watt balances and the Avogadro constant determined by the XRCD method. Closure of the triangle within the claimed uncertainties of the various input data would be evidence that all methods are well understood (Fig. 2a). The target uncertainties for the watt balance and XRCD experiments are shown, as is the present uncertainty of $N_A h$. The present uncertainties for the watt balance and the XRCD method are higher than these targets. More worrisome, however, is the fact that, even at the present uncertainties, the triangle does not close (Fig. 2b). A weighted mean of all results (dominated by the most recent NIST watt balance result) is not consistent with the XRCD result. The Avogadro experiment is being repeated with different material which will permit a smaller uncertainty. The result will, perhaps, elucidate some unexpected error within the triangle. The NPL watt balance has produced a recent result, not included in the CODATA



a

b

Fig. 2. (a) Ideally, the international prototype (IPK) will be linked to the Avogadro constant through the XRCD method and to the Planck constant by watt balances, all with a relative standard uncertainty of about 20×10^{-9}. However, the link between the Avogadro and Planck constants is already known to much smaller uncertainty so that the XRCD method can also link the international prototype to the Planck constant, or the watt balance to the Avogadro constant. (b) At present, the triangle does not close

2006 adjustment but mentioned in the text [8], which is between the NIST and XRCD measurements. Thus the triangle shown in Fig. 2 is somewhat idealized and oversimplified. This ongoing work is important to the practical realization of a kilogram redefined in terms of a fundamental constant, such as h.

5. The push to redefine the kilogram, ampere, the kelvin and the mole

In response to proposals to redefine the kilogram, ampere, kelvin and mole in terms of fundamental constants, the International Committee for Weights and Measures (CIPM) in 2005 called upon their relevant consultative committees to initiate certain preparatory steps that might lead to redefinition of the kilogram and other base units [7]. The recommendations of the CCM were given in a subsequent report to the CIPM [12]. These call attention to the need to have several on-going experiments capable of realizing the new definition to an accuracy of order 20 µg/kg, the need to resolve the discrepancies in measurements of the same constant using watt balances and the XRCD method and the importance of formulating an agreed method for realizing the kilogram in accordance with any new definition. Such a method is referred to by its French name, *mise en pratique*. In light of the discussion given above in section 3.3, the CCM did not express a preference for fixing a particular constant in order to redefine the kilogram, so long as a practical link to a macroscopic kilogram could be made.

The SI has evolved throughout its history. We may take as an example the redefinition of the metre in terms of the second in 1983. This definition implicitly fixed the value of the speed of light. Until then, the speed of light was a fundamental constant whose measured value and uncertainty both changed as experiments improved. After the redefinition, to measure the speed of light became a logical impossibility-its value is defined. The same logic applies to fixing a value of the Planck constant. The day before such a definition takes effect, a watt balance or the XRCD method determines the value of h with an evaluated uncertainty. The day after the value of h is fixed, the same experiments give us the value of the IPK to the same uncertainty.

It seems clear that if and when the kilogram is redefined, the realization by a particular apparatus will not be as accurate as the precision with which 1 kg mass standards are commonly compared. In view of this situation, the CCM has organized a Task Group to anticipate any operational difficulties and to deal with them efficiently.

At the most recent of its quadrennial meetings

(2007), the CGPM passed Resolution 12 "On the possible redefinition of certain base units of the International System of Units (SI), which opens the possibility of approving redefinitions of the kg, A, K and mol at its next meeting, in 2011. The resolution is publically available on the BIPM web site. However, it is understood that a certain number of outstanding problems must first be resolved, including a satisfactory understanding of the present significant disagreement between results obtained by watt balances and the XRCD method [13].

6. Conclusion

Upon the completion of the third verification (1992), the CCM called for NMIs to establish experimental links between mass standards and fundamental constants with a view toward an eventual redefinition of the unit of mass. This request was endorsed by the CGPM in 1995. The perfection of quantum electrical standards has had the inevitable consequence that electrical metrologists wish to assign fixed values to the Planck constant and the elementary charge without delay. Defining a fixed value for the Planck constant inevitably redefines the kilogram. Thus the mass of the present artifact definition would acquire an experimental uncertainty, although there would be no discontinuity its value, 1 kg.

The additional uncertainty would affect all macroscopic mass measurements in the same way and thus could be ignored except in very unusual circumstances. Nevertheless, this will add a new complication to the mass metrology of macroscopic objects. However, there will also be a benefit to mass metrology: provided that watt balances and the XRCD method are maintained indefinitely, we will have a robust system in place to ensure that, in the long term, macroscopic masses stay linked to the fundamental constants to within known limits.

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